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AUGUST 1971 AIAA JOURNAL VOL. 9, NO. 8

Optimization of Complex Structures to Satisfy Flutter Requirements

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Equations for finding the partial derivatives of the flutter velocity of an aircraft structure with respect to structural parameters are derived. A numerical procedure is developed for determining the values of the structural parameters such that a specified flutter velocity constraint is satisfied and the structural mass is a relative minimum. A search procedure is presented which utilizes two gradient search methods and a gradient projection method. The procedure is applied to the design of a box beam.

I. Introduction

STRUCTURAL design is often accomplished through a series of design iterations, in which a trial design is chosen, analyzed and modified by the designer after examination of the numerical results. This iterative process does not guarantee a design of minimum weight for all design conditions since the designer's judgment and intuition are influencing factors in the redesign process and only a small number of design iterations are practical. Even with the aid of the digital computer, the design of complex structures to satisfy dynamic response restrictions has been hampered by the inherent difficulty and the computational cost of dynamic analysis. Recent advances in programing techniques, and matrix methods of structural analysis have provided all the necessary tools for the development of efficient structural optimization methods.

During the last few years numerous publications have appeared in the literature[1–4] dealing with the dynamic optimization of structures. M. J. Turner[1] developed a procedure for determining the relative proportions of selected elements of an aircraft structure to attain a specified flutter speed with minimum total mass. Lagrange multipliers were employed to introduce the conditions for neutral dynamic stability (flutter equations) as dynamic constraints. The resulting system of nonlinear equations was solved by the Newton-Raphson process to determine the masses of the elements of the system.

McCart, Haug and Streeter[2] developed a steepest-descent boundary-value method for the design of structures with constraints on strength and natural frequency. A computational algorithm was developed which implemented the steepest-descent method. The method was developed in detail for a three-member frame design problem and references were given for a more general development.

Fox and Kapoor[3] developed a structural optimization procedure in which limitations were imposed upon maximum dynamic stresses and displacements (handled by the shock spectral approach) as well as the natural frequencies of the structure. A direct optimization method (the method of feasible directions) which consisted of a design-analysis cycle was used. Exact and computationally efficient schemes were developed for finding derivatives of maximum response and natural frequencies.

C. P. Rubin[4] developed a procedure for the determination of the least weight structure for a specified natural frequency requirement. The method modified an existing structure by varying the cross-sectional properties of its members. This was accomplished by using gradient equations to first obtain the desired structural frequency, and then separate gradient equations were used to decrease the weight of the structure while the natural frequency was held constant.

Most of the important papers dealing with the subject of optimum design of structures for dynamic requirements are listed as references in the works cited. The methods used in Refs.[1–4] appear to be most promising and improvement and extension of these methods to include aeroelastic constraints such as divergence speed, reversal of control speed and flutter speed appears feasible. In principle there are many ways in which an optimum design of a structure may be found; essentially the problem is that of developing an efficient design procedure which requires a reasonable amount of computer storage and run time to find the optimum design of a complex structure.

II. Description of Optimization Procedure

The primary objective of this paper is to develop a numerical procedure for determining a selected number of structural dimensions or parameters for the optimum (minimum mass) design of a complex structure with a specified flutter velocity. Structural parameters (cross-sectional areas, plate

Received December 21, 1970. The authors wish to express their appreciation to R. V. Doggett, Jr. of the NASA Langley Research Center for his many valuable suggestions during the course of this work. This research was supported by NASA Research Grant NGR-41-001-027.

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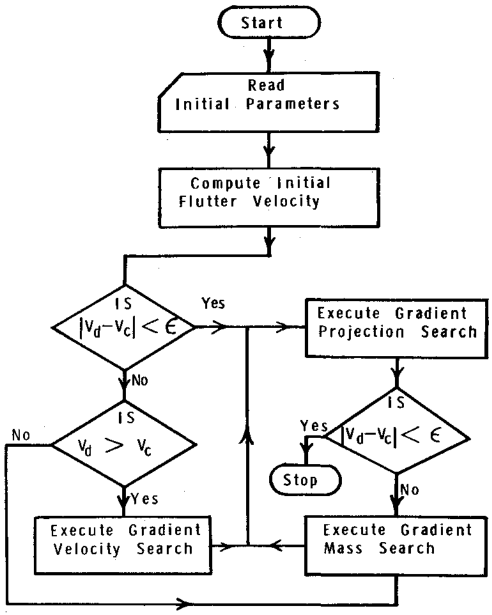

Fig. 1 Simplified flow diagram for optimization procedure.

thicknesses, diameters squared, etc.) are selected in such a way that the total mass of the structure is a linear function of these parameters.

The optimization procedure developed here is independent of the aerodynamic theory selected and the method of formulating the flutter equations. The finite-element methods may be employed for optimizing simple structures by using nodal displacements as generalized coordinates in formulating the flutter equations. For complex structures the flutter equations may be formulated by introducing generalized model coordinates which are associated with the displacement shapes. The optimization procedure utilizes three search routines that are described below.

A. Velocity Gradient Search

This routine is employed when it is desired to increase the flutter velocity. The flutter velocity normal derivative is calculated at a point and parameters are varied so that a step is taken in the direction of maximum increase in velocity. The desired flutter velocity is reached after several successive steps along a gradient curve in an iterative fashion.

B. Gradient Mass Search

This routine is employed whenever it is desired to reduce the flutter velocity. The normal derivative of the total structural mass is computed at a point. A step is taken in the direction of the greatest rate of decrease in the structural mass. The process is repeated until the flutter velocity is less than or equal to the desired value.

C. Gradient Projection Search

This routine is employed to find a relative maximum of the flutter velocity while the total mass of the structure is held constant. The parameters are varied in such a way that the search proceeds tangent to a constant mass hyperplane in the direction of the maximum rate of increase of the flutter velocity until a relative maximum is found which lies within the bounds of the parameter constraints.

A simplified flow diagram of the optimization procedure is shown in Fig. 1. The desired flutter velocity is V_d, the computed flutter velocity is V_c and the velocity tolerance is ϵ. If V_c is within tolerance then a gradient projection search is executed along a constant mass hyperplane. The maximum velocity found in this search will be greater than or equal to the $V_d - \epsilon$. If the new velocity is within tolerance then the parameters are a relative optimum set. If the velocity is not within tolerance then it is decreased along the negative mass gradient direction until it is within tolerance. The process is repeated in an iterative fashion until V_c is within tolerance after leaving the gradient projection search.

If the flutter velocity V_c is not within tolerance and is less than V_d, then the velocity gradient search is executed instead of the mass gradient search. The velocity gradient search will increase the flutter velocity along a path which produces the maximum rate of increase of the flutter velocity.

III. Partial Derivatives of the Flutter Velocity

Consider an aircraft structure which is undergoing steady-state oscillations and is in a state of neutral stability. The equation of oscillation for the structure may be expressed in one of the forms

$$[[K] - \lambda([M] + [A])]\{U\} = 0 \tag{1}$$

$$[[C]([M] + [A]) - \bar{\lambda}[I]]\{U\} = 0 \tag{1a}$$

where $\{U\}$ = column vector of generalized coordinates; $[K]$ = stiffness matrix; $[M]$ = inertia matrix; $[A]$ = air-force matrix (function of reduced frequency k, air density, Mach number and semichord), complex; $[C]$ = flexibility matrix; $[I]$ = identity matrix; $k = b\omega/V_f$; $\lambda = \omega^2$ = eigenvalue; $\bar{\lambda} = 1/\omega^2$; ω = oscillation frequency, rad/sec; b = semichord; and V_f = flutter velocity.

The stiffness, flexibility and inertia matrices may be expressed in the forms:

$$[K] = [K]_c + [K]_v \tag{2}$$

$$[C] = [C]_c + [C]_v \tag{2a}$$

and

$$[M] = [M_c] + [M]_v \tag{2b}$$

where $[K]_c$, $[C]_c$ and $[M]_c$ are constant matrices whereas $[K]_v$, $[C]_v$ and $[M]_v$ are matrices which are functions of the variable structural parameters P_i.

If the eigenvectors in Eq. (1) exist, it can be shown that there also exist eigenvectors $\{V\}$ such that

$$\{V\}^T[[K] - \lambda([M] + [A])] = 0 \tag{3}$$

and the eigenvalues of Eq. (3) are identical to those of Eq. (1). The row matrix $\{V\}^T$ is the transpose of $\{V\}$ and will be referred to as the associated row vector of the eigenvector $\{U\}$.

The characteristic Eq. (1) may be differentiated with respect to the parameters P_i, which yields

$$\left[\frac{\partial[K]_v}{\partial P_i} - \frac{\partial\lambda}{\partial P_i}([M] + [A]) - \lambda\left(\frac{\partial[M]_v}{\partial P_i} + \frac{\partial[A]}{\partial k}\frac{\partial k}{\partial P_i}\right)\right] \times \{U\} + [[K] - \lambda([M] + [A])]\frac{\partial\{U\}}{\partial P_i} = 0 \tag{4}$$

Premultiplying Eq. (4) by the associated row vector $\{V\}^T$ and substituting Eq. (3) into the resulting equation yields after some manipulation the partial derivatives of the eigenvalues with respect to the parameters P_i

$$\frac{\partial\lambda}{\partial P_i} = \left[\{V\}^T\left(\frac{\partial[K]_v}{\partial P_i} - \lambda\frac{\partial[M]_v}{\partial P_i}\right)\{U\} - \lambda\{V\}^T\frac{\partial[A]}{\partial k} \times \{U\}\frac{\partial k}{\partial P_i}\right] \Big/ \{V\}^T([M] + [A])\{U\} \tag{5}$$

The elements of the matrices $[A]$, $\{U\}$ and $\{V\}$ may be real or complex but we are interested only in the steady-state oscillations of the system. When the system flutters, then λ and k are real, hence $\partial\lambda/\partial P_i$ and $\partial k/\partial P_i$ must be real if the system is to move from one flutter velocity to another flutter velocity with each set of changes in the real parameters P_i. To assure that $\partial\lambda/\partial P_i$ and $\partial k/\partial P_i$ are real let

$$\frac{\partial\lambda}{\partial P_i} = \frac{R_1 + I_1 i - (R_2 + I_2 i)\partial k/\partial P_i}{R_3 + I_3 i} \tag{6}$$

where

$$R_1 + I_1 i = \{V\}^T(\partial[K]_v/\partial P_i - \lambda\partial[M]_v/\partial P_i)\{U\} \tag{7}$$

$$R_2 + I_2 i = \lambda\{V\}^T(\partial[A]/\partial k)\{U\} \tag{8}$$

$$R_3 + I_3 i = \{V\}^T([M] + [A])\{U\} \tag{9}$$

and

$$i = (-1)^{1/2}$$

Multiplying numerator and denominator of Eq. (6) by $R_3 - I_3 i$ and then separating real and imaginary parts produces the relation

$$\frac{\partial\lambda}{\partial P_i} = \left\{\frac{(R_1 - R_2\partial k/\partial P_i)R_3 + (I_1 - I_2\partial k/\partial P_i)I_3}{R_3^2 + I_3^2}\right\} + \left\{\frac{(I_1 - I_2)(\partial k/\partial P_i)R_3 - (R_1 - R_2\partial k/\partial P_i)I_3}{R_3^2 + I_3^2}\right\} i \tag{10}$$

If $\partial\lambda/\partial P_i$ is real then the second member on the right of Eq. (10) must vanish, i.e.,

$$(I_1 - I_2\partial k/\partial P_i)R_3 = (R_1 - R_2\partial k/\partial P_i)I_3$$

hence

$$\partial k/\partial P_i = (R_1 I_3 - I_1 R_3)/(R_2 I_3 - I_2 R_3) \tag{11}$$

and

$$\frac{\partial\lambda}{\partial P_i} = \frac{(R_1 - R_2\partial k/\partial P_i)R_3 + (I_1 - I_2\partial k/\partial P_i)I_3}{R_3^2 + I_3^2} \tag{12}$$

If Eq. (11) is satisfied then $\partial k/\partial P_i$ and $\partial\lambda/\partial P_i$ will be pure real variables.

In a similar manner, starting with Eq. (1a), it can be shown that

$$R_1 + I_1 i = \{V\}^T \left\{\frac{\partial[C]_v}{\partial P_i}([M] + [A]) + [C]\frac{\partial[M]_v}{\partial P_i}\right\}\{U\} \tag{13}$$

$$R_2 + I_2 i = \{V\}^T([C]\partial[A]/\partial k)\{U\} \tag{14}$$

and

$$R_3 + I_3 i = \{V\}^T\{U\} \tag{15}$$

The partial derivatives of the velocity with respect to the parameters may be found by considering the definition of the reduced frequency

$$k = b\omega/V_f \tag{16}$$

where V_f is the flutter velocity (velocity at a neutral stable state). The partial derivative of V_f may be expressed by the relation

$$\partial V_f/\partial P_i = -b\omega/k^2(\partial k/\partial P_i) + b/2k\omega(\partial\lambda/\partial P_i) \tag{17}$$

or

$$\partial V_f/\partial P_i = -b\omega/k^2(\partial k/\partial P_i) - b\omega^3/2k(\partial\bar{\lambda}/\partial P_i) \tag{18}$$

The partial derivatives of the flutter velocity in expressions (17) and (18) will be pure real variables if Eqs. (11) and (12) are satisfied.

IV. Velocity Gradient Search

The maximum rate of change of the flutter velocity is given by the normal derivative and is equal to the absolute value of the gradient vector of the flutter velocity; it may be expressed as

$$\frac{dV_f}{dS} = \left[\sum_{i=1}^{n}\left(\frac{\partial V_f}{\partial P_i}\right)^2\right]^{1/2} \tag{19}$$

A new velocity along a gradient curve corresponding to small parameter changes may be approximated by the expression

$$V_f^* \simeq V_f + (dV_f/dS)\Delta S \tag{20}$$

where V_f^* and V_f are the new and old flutter velocities, respectively.

The directional cosines of the gradient are

$$l_i = \frac{\partial V_f}{\partial P_i}\Big/\left[\sum_{i=1}^{n}\left(\frac{\partial V_f}{\partial P_i}\right)^2\right]^{1/2} \simeq \frac{\Delta P_i}{\Delta S} \tag{21}$$

where

$$(\Delta S)^2 = \sum_{i=1}^{n}(\Delta P_i)^2$$

hence

$$\Delta P_i \simeq \frac{\partial V_f}{\partial P_i}\Delta S\Big/\left[\sum_{i=1}^{n}\left(\frac{\partial V_f}{\partial P_i}\right)^2\right]^{1/2} \tag{22}$$

From Eqs. (19, 20 and 22) the following equation for ΔP_i may be derived:

$$\Delta P_i \simeq \frac{\partial V_f}{\partial P_i}(V_f^* - V_f)\Big/\sum_{i=1}^{n}\left(\frac{\partial V_f}{\partial P_i}\right)^2 \tag{23}$$

New values of the parameters P_i^* may be computed from the relations

$$P_i^* = P_i + \Delta P_i \tag{24}$$

by substituting values of $\partial V_f/\partial P_i$ from Eq. (17) or (18) along with V_f^* and V_f into Eq. (23) and solving for P_i^* from Eq. (24). A new value of the reduced frequency may be estimated from the relation

$$k^* = k + \sum_{i=1}^{n}(\partial k/\partial P_i)\Delta P_i \tag{25}$$

where values of $\partial k/\partial P_i$ are determined from Eq. (11). This new value of k may be used as a starting value for determining the true flutter velocity from Eq. (1) or (1a) for values of P_i^*.

Since the flutter velocity is a nonlinear function of the parameters, the true flutter velocity corresponding to the new set of parameters P_i^* will not equal V_f^*. The correct flutter velocity must be determined from the characteristic Eq. (1) or (1a) using values of P_i^*; the process may be repeated in an iterative fashion until parameters P_i^* are found which correspond to the desired flutter velocity. The search procedure used here is similar to the "Method of Steepest Ascent" as described in Ref. 5.

V. Mass Gradient Search

The total derivative of the flutter velocity may be expressed as

$$dV_f = \sum_{i=1}^{n}(\partial V_f/\partial P_i)dP_i \tag{26}$$

The normal derivative of the total mass m of a structure may be expressed by the equation

$$\frac{dm}{dS} = \left[\sum_{i=1}^{n}\left(\frac{\partial m}{\partial P_i}\right)^2\right]^{1/2} \tag{27}$$

and the direction cosines of the mass gradient vector are given

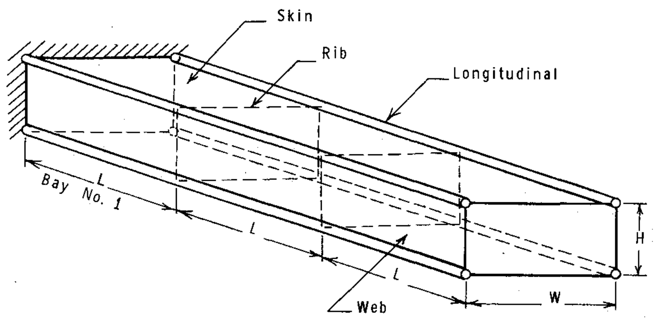

Fig. 2 Rectangular box beam.

by the relations

$$\frac{\partial m}{\partial P_i}\Bigg/\left[\sum_{i=1}^{n}\left(\frac{\partial m}{\partial P_i}\right)^2\right]^{1/2} \simeq \frac{\Delta P_i}{\Delta S} \tag{28}$$

where

$$(\Delta S)^2 = \sum_{i=1}^{n} (\Delta P_i)^2$$

Substituting ΔP_i from Eq. (28) into Eq. (26) and setting $\Delta V_f \simeq dV_f$ yields the relation

$$\Delta S \simeq \left[\sum_{i=1}^{n}\left(\frac{\partial m}{\partial P_i}\right)^2\right]^{1/2} \Delta V_f \Bigg/ \sum_{i=1}^{n} \frac{\partial V_f}{\partial P_i}\frac{\partial m}{\partial P_i} \tag{29}$$

After substituting Eq. (29) into Eq. (28) and solving for ΔP_i it is seen that

$$\Delta P_i \simeq \frac{\partial m}{\partial P_i}\Delta V_f \Bigg/ \sum_{i=1}^{n} \frac{\partial V_f}{\partial P_i}\frac{\partial m}{\partial P_i} \tag{30}$$

where $\Delta V_f = V_f^* - V_f$ and $P_i^* = P_i + \Delta P_i$.

As in the velocity gradient search the mass gradient search is an iterative process. The mass gradient search is used when it is desired to decrease the flutter velocity. Some of the values of ΔP_i may be positive; when this occurs the corresponding P_i is held constant and the search will deviate from the gradient direction. This assures that the mass of the structure will not increase during the search.

VI. Gradient Projection Search

The gradient projection method allows variations of parameters to be taken in such a way that constraints are automatically satisfied while the search for an optimum proceeds in the direction of steepest ascent. Here the flutter velocity will be maximized while the total structural mass is held constant, then

$$dm = \sum_{i=1}^{n} (\partial m/\partial P_i)dP_i \tag{31}$$

where m is the mass of the entire structure. Equation (31) is a functional constraint which requires the search to move along a constant mass hyperplane since the variable parameters P_i are selected in such a way that the mass is a linear function of the parameters.

By using the method of Lagrangian multipliers it can be shown that the maximum rate of increase in the flutter velocity along a constant mass hyperplane is given by the relation (see Ref. 6 for details of the mathematical development):

$$\frac{dV_f}{dS} = \left\{\sum_{i=1}^{n}\left[\left(\frac{\partial V_f}{\partial P_i}\right)^2 + \frac{\partial V_f}{\partial P_i}\lambda_1\frac{\partial m}{\partial P_i}\right]\right\}^{1/2} = 2\lambda_0 \tag{32}$$

where λ_0 and λ_1 are Lagrangian multipliers and

$$\lambda_1 = -\sum_{j=1}^{n}\frac{\partial m}{\partial P_j}\frac{\partial V_f}{\partial P_j}\Bigg/\sum_{j=1}^{n}\left(\frac{\partial m}{\partial P_j}\right)^2 \tag{33}$$

A new set of parameters can be determined from the relation

$$P_i^* = P_i + 1/2\lambda_0(\partial V_f/\partial P_i + \lambda_1 \partial m/\partial P_i)S \tag{34}$$

where S is a sufficiently small step size tangent to the constant mass hyperplane. The value of S is determined by trial; if a selected value of S results in an increased value of V_f then S is increased for the next trial calculation; if V_f decreases S is decreased. When λ_0 becomes small then V_f is near a maximum for the constant mass hyperplane.

The gradient projection method can be easily modified so that a constant flutter velocity hypersurface can be followed in the direction of the maximum rate of decreasing mass along the hypersurface by interchanging m and V_f in Eqs. (32–34) and by placing a negative sign before the radical of Eq. (32). Since the constant hypersurface is nonlinear, the gradient search will step off of the hypersurface. When excessive drift of the flutter velocity away from the desired value occurs, the gradient velocity search or the gradient mass search may be used to get back to the desired constant flutter hypersurface.

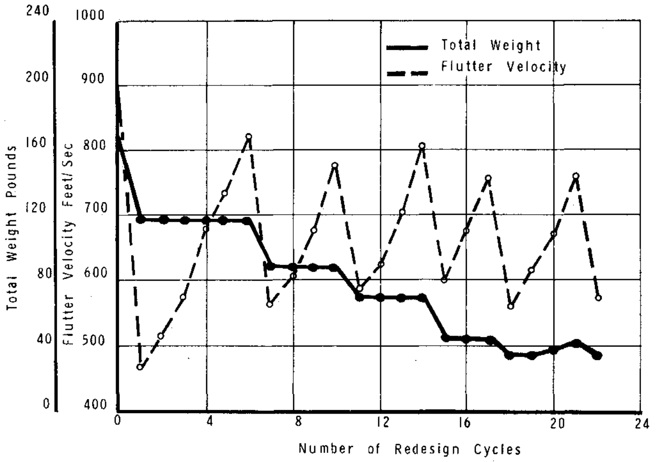

Fig. 3 Computed flutter velocity and total weight of beam for 600 fps flutter velocity constraint.

VII. Design of a Box Beam Spar

The three search procedures (velocity gradient search, mass gradient search and gradient projection search) and the over-all optimization procedure (see Fig. 2) were programed in such a way that arbitrary flutter analysis subroutines and structural analysis subroutines could be called. The procedure was applied to the design of the box beam shown in Fig. 2. The beam supports a rectangular lifting surface whose cross section is uniform. The initial variable parameters of the beam are given in Table 1. Other constant dimensions of the beam and parameters needed in the flutter analysis were assumed to be: $L = 5$ ft (see Fig. 2); $H = 4$ in. (see Fig. 2); $W = 25$ in. (see Fig. 2); $\rho = 5.46$ slugs/ft^3, density of spar material; $E = 10.0 \times 10^6$ psi, modulus of elasticity; $G = 4.0 \times 10^6$ psi, modulus of rigidity; $b = 25$ in., semichord; $a = 2.5$ in., distance of elastic axis upstream from the midchord; and $V_f = 600$ fps at 10,000 ft.

Table 1 Initial variable parameters of box beam

Bay no.	Areas of longitudinals	Front and back web thicknesses	Top and bottom skin thickness	Rib thickness
1	2.0 in.^{2}	0.08 in.	0.04 in.	0.04 in.
2	2.0 in.^{2}	0.08 in.	0.04 in.	0.04 in.
3	2.0 in.^{2}	0.08 in.	0.04 in.	0.04 in.

Table 2 Final variable parameters of box beam

Bay no.	Areas of longitudinals	Front & back web thicknesses	Top & bottom skin thickness	Rib thickness
1	0.333 in.^{2a}	0.0587 in.	0.00666 in.[a]	0.0365 in.
2	0.333 in.^{2a}	0.0534 in.	0.00953 in.	0.0338 in.
3	0.333 in.^{2a}	0.0501 in.	0.00666 in.[a]	0.0335 in.

[a] Minimum permissible values.

For convenience the mass and stiffness of the leading and trailing edges of the lifting surface were neglected, i.e., only the mass and stiffness of the beam were considered. The elements of the air-force matrix were computed from formulas given in Ref. 7.

The final parameters found by the optimization procedure are shown in Table 2. Arbitrary constraints were placed on the variable parameters, minimum values were set at one-sixth of the initial values.

The gradient projection search was modified in such a way that if the computed flutter velocity exceeded the desired flutter velocity by 25%, then the mass gradient search was called. The search then tends to follow a constant velocity hypersurface until a relative minimum mass structure is found.

The total weight of the beam and the flutter velocity for each iteration were plotted versus the number of redesign cycles in Fig. 3. The velocity was first decreased by the mass gradient search; this was followed by the gradient projection search in which the mass was held constant. When the computed flutter velocity exceeded the desired value by 25% or more, the mass gradient search was again executed. The procedure was repeated in an iterative fashion until the mass would no longer decrease by a significant amount. Toward the end of the solution the mass would increase slightly during the gradient projection search; this was because some of the parameters had reached specified minimum values and could not be decreased further while other parameters could be decreased or increased. It may be possible to circumvent this effect by excluding from the computations those parameters which have reached extreme values. The solution was stopped after the cross-sectional areas of the longitudinals reached the pre-imposed minimum values and the mass would no longer decrease by a significant value. The total CPU (central processing unit) time on an IBM 360 Model 50 digital computer was 15.677 min.

The desired flutter velocity was changed to 800 fps and the computations were resumed. Plots of flutter velocity and total weight for the 800 fps constraint were very similar to those in Fig. 3, except the first two redesign cycles utilized the velocity gradient search to find the constant velocity hypersurface. The velocity gradient search rather than the mass gradient search is used to increase the velocity since it would tend to produce less increase in the total mass.

VIII. Conclusion

The analytical derivation of expressions for the derivatives $\partial\lambda/\partial P_i$, $\partial k/\partial P_i$ and $dV_f/\partial P_i$ is regarded as the most important feature of this paper. Since computation of these derivatives is essential to gradient type optimization procedures, these expressions (for the derivatives) form the basis for the "practical" optimum design procedure developed here. It is important to note that this procedure can simultaneously vary all the parameters to be optimized, as dictated by the search procedures. Also the treatment presented here would lend itself to incorporation of additional constraints, which the authors hope to treat in their future work.

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